A MATHEMATICAL MODEL OF HEAT TRANSFER FOR FORCED-CONVECTIVE FILM BOILING

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UDC 536.24

A mathematical model of film boiling with forced convection of the liquid is discussed. A solution is obtained on a computer and is represented in the form of nomographs and approximation relations. The computed results are compared with experimental data.

There are three possible regimes associated with forced-convective film boiling: 1) annular- (or core-) flow; 2) slug-flow; 3) mist- (dispersed-, spray-) flow [1, 2]. The present study is concerned with the annular-flow regime of film boiling in a pipe, where the liquid core is separated from the wall by an annular vapor film.

We consider a one-dimensional steady-state model of film boiling with liquid flow in a circular pipe having an arbitrary lengthwise distribution of the wall temperature (Fig. 1). The liquid moves in the form of a turbulent jet, which is separated from the pipe wall by a turbulent vapor film. The phase interface is assumed to be smooth and axisymmetrical. The interfacial temperature is equal to the saturation temperature T_S; i.e., the rate of the phase-transition processes is assumed to be infinitely great, and the probability of metastable thermodynamic states to be negligibly small.

The heat flux is directed from the wall to the phase interface through the vapor film due to turbulent heat transfer and results in heating of the liquid (if $T_L < T_S$) along with evaporation.

We disregard pressure variation along the pipe, longitudinal heat conduction, dissipation, and radiative energy transfer.

In addition to these customary constraints, we adopt the following assumptions:

1. Interphase slip does not occur ($U_V = U_L = U$); this assumption is valid for large flow velocities and small pressure differentials along the pipe, such that $U_V/U_L \approx 1$.

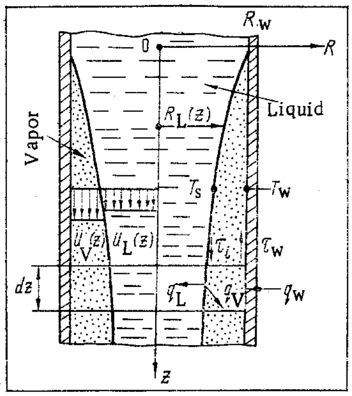

Fig. 1. Schematic diagram of film boiling in a pipe.

S. Ordzhonikidze Moscow Aviation Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 33, No. 5, pp. 822-827, November, 1977. Original article submitted January 24, 1977.

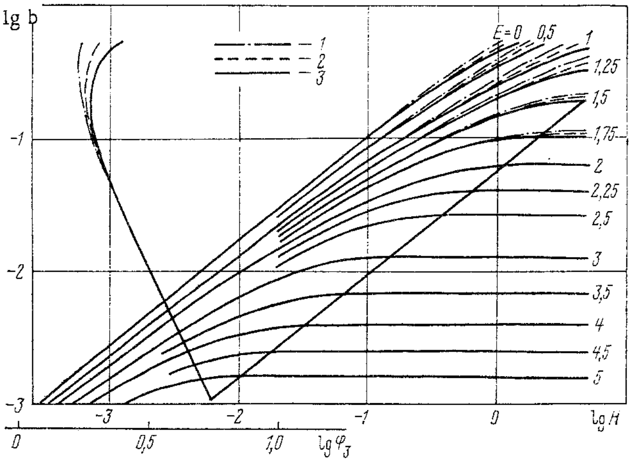

Fig. 2. Nomograph for calculation of the dimensionless film thickness, m = 0.75, n = 1. 1) ρ_V/ρ_L = 0.2; 2) 0.1; 3) 0.

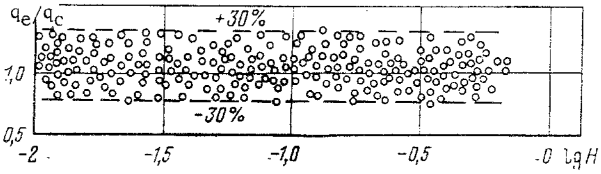

Fig. 3. Comparison of experimental and computed heat-flux values.

2. The bulk-mean temperature of the vapor is equal to the saturation temperature T_S; i.e., it is assumed that the main thermal resistance of the vapor film is concentrated in a narrow domain close to the pipe wall.

3. The heat transfer from the wall into the vapor film obeys the law

$$\mathrm{Nu}_V = A\,\mathrm{Re}_V^m, \tag{1}$$

where, by analogy with an annular duct, twice the thickness of the vapor film is adopted as the space scale.

4. The heat flux q_L in the liquid does not depend on the vapor content and is determined solely by the process of turbulent heat conduction in the liquid jet. The heat transfer into the liquid obeys the law

$$\mathrm{Nu}_L = B\,\mathrm{Re}_L^n \tag{2}$$

in which the diameter of the liquid core, $D_L = D - 2\delta$, is adopted as the space scale.

With the foregoing assumptions in mind, we write the continuity and energy equations in the form

$$\rho_V U\pi(D-\delta)\delta + \rho_L U\frac{\pi}{4}(D-2\delta)^2 = \rho_L U_0\frac{\pi D^2}{4}; \tag{3}$$

$$\pi D\frac{\lambda_V}{2\delta}(T_W - T_s)A\left(\frac{U2\delta}{\nu_V}\right)^m = \frac{\lambda_L}{D-2\delta}(T_s - T_L)B\left(U\frac{D-2\delta}{\nu_L}\right)^n\pi(D-2\delta) + r\frac{d}{dz}[\rho_V U\pi(D-\delta)\delta]. \tag{4}$$

Determining the two-phase flow velocity U from (3) and inserting it into (4), we obtain a differential equation describing the variation of the film thickness along the pipe

$$\rho_V U_0 r\frac{d}{dz}\left[\frac{(D-\delta)\delta}{\frac{\rho_V}{\rho_L} + \left(1-\frac{\rho_V}{\rho_L}\right)\left(1-\frac{2\delta}{D}\right)^2}\right] =$$

$$= \frac{\lambda_V (T_W - T_s) A (U_0 D/\nu_V)^m}{\left(\frac{2\delta}{D}\right)^{1-m} \left[\frac{\rho_V}{\rho_L} + \left(1 - \frac{\rho_V}{\rho_L}\right)\left(1 - \frac{2\delta}{D}\right)^2\right]^m} -$$

$$- \frac{\lambda_L (T_s - T_L) B (U_0 D/\nu_L)^n \left(1 - \frac{2\delta}{D}\right)^n}{\left[\frac{\rho_V}{\rho_L} + \left(1 + \frac{\rho_V}{\rho_L}\right)\left(1 - \frac{2\delta}{D}\right)^2\right]^n} \tag{5}$$

subject to the initial condition

$$z = 0; \quad \delta = 0. \tag{6}$$

We introduce the dimensionless governing parameters

$$H = A \left(\frac{U_0 D}{\nu_n}\right)^m \frac{4\lambda_V}{\rho_V U_0 r D^2} \int_0^z (T_W - T_s)\, dz; \tag{7}$$

$$E = \frac{\lambda_L (T_s - T_L)}{\lambda_V (T_W - T_s)} \cdot \frac{B (U_0 D/\nu_L)^n}{A (U_0 D/\nu_n)^m}. \tag{8}$$

Now Eq. (5) is transformed as follows:

$$\frac{db}{dH} = \varphi_1(b) - E\varphi_2(b), \tag{9}$$

where $b = 2\delta/D$ is the dimensionless thickness of the vapor film and

$$\varphi_1(b) = \frac{\left[(1-b)^2 + \frac{\rho_V}{\rho_L}(2-b)\,b\right]^{2-m}}{2b^{1-m}(1-b)}; \tag{10}$$

$$\varphi_2(b) = \frac{\left[(1-b)^2 + \frac{\rho_V}{\delta_L}(2-b)\,b\right]^{2-n}}{2(1-b)^{1-n}}. \tag{11}$$

If E does not depend on H, the differential equation (9) is reduced to the integral

$$H = \int_0^b \frac{db}{\varphi_1(b) - E\varphi_2(b)}. \tag{12}$$

In the general case, E depends on H, because the parameter H is a monotonically increasing function of the z coordinate and the dimensionless group E includes the quantities B and $(T_W - T_S)$, which in general also depend on z. However, computer calculations related to the numerical integration of Eq. (9) show that the dimensionless film thickness in a given cross section is determined only by the values of the parameters H and E in that cross section. This result can be explained physically as follows. If the fraction of heat that goes into the liquid (E) is sufficiently large, then the prior history of the flow (H) has little influence on the heat-transfer process, because in each cross section the vapor film acquires a thickness such that the heat flux required by the liquid is in fact transmitted. With an increase in the fraction of heat used for evaporation, the influence of the flow history (H) on the heat-transfer process and film thickness becomes significant, but in return the dependence of these factors on the fraction of heat transmitted into the liquid (E) abates.

Expression (12) enables us to determine the dimensionless vapor-film thickness b as a function of the dimensionless parameters H, E, ρ_V/ρ_L and to compute the heat flux according to the equation

$$q_W = \frac{\lambda_V (T_W - T_S)}{D} A \left(\frac{U_0 D}{\nu_V}\right)^m \varphi_3(b), \tag{13}$$

in which

$$\varphi_3(b) = b^{m-1}\left[(1-b)^2 + \frac{\rho_V}{\rho_L}(2-b)\,b\right]^{-n}. \tag{14}$$

We integrated expression (12) numerically on an M-220 digital computer for the case $m = n = 0.8$. The computational results are displayed in the form of a nomograph, which permits the values of b and φ_3 (b) to be determined. We also approximate expression (12) in the ranges $H = 10^{-3}$ to 5, $E = 0$ to 4, $\rho_V/\rho_L = 0$ to 0.2, $b \leq 0.5$ by the equation

$$b = \{[1.53H^{-5/6} + 0.94(1 - \rho_V/\rho_L)]\exp(0.2E)[E - 0.12(1 - \rho_V/\rho_L)\exp(-0.26E)]^5\}^{-1}. \tag{15}$$

To compute the parameters H, E and the heat flux q_W it is necessary to assign a specific form to Eqs. (1) and (2).

We assume that the heat flux carried from the wall by the vapor can be calculated according to the equation for heat transfer to a gas in an annular duct [3]

$$\mathrm{Nu}_V = A\,\mathrm{Re}_V^{0.8} = 0.0198\,\mathrm{Re}_V^{0.8}\,\mathrm{Pr}_V^{0.4}\left(\frac{T_w}{T_s}\right)^{-0.55}, \tag{16}$$

and the heat flux into the liquid according to the well-known expression for heat transfer into a liquid in a circular pipe

$$\mathrm{Nu}_L = B\,\mathrm{Re}_L^{0.8} = 0.023\,\mathrm{Re}_L^{0.8}\,\mathrm{Pr}_L^{0.4}\,f(z/D). \tag{17}$$

We compare the heat fluxes calculated in this way with our earlier experimental data [4]. The comparison of the experimental and computed results shows that the computation affords valid order-of-magnitude estimates of the heat fluxes, albeit with rather large scatter (± 50%). It is important to bear in mind, however, that the computations are based solely on the standard empirical formulas for single-phase media.

The computational error can be substantially reduced by using the empirical film-boiling heat-transfer relations proposed in [4] in the role of relations (1) and (2)

$$\mathrm{Nu}_V = A\,\mathrm{Re}_V^{0.75} = 0.0078\,\mathrm{Re}_V^{0.75}\,\mathrm{Pr}_V[1 + 3\cdot10^{-6}\psi\,1.3\exp(-1\cdot10^{-6}\psi)]; \tag{18}$$

$$\mathrm{Nu}_L = B\,\mathrm{Re}_L = 0.0012\,\mathrm{Re}_L\,\mathrm{Pr}_L^{0.4}\,f_1(z/D). \tag{19}$$

The results of numerical integration of (12) for the case $m = 0.75$, $n = 1$ are given in nomograph form in Fig. 2 and are approximated by the expression

$$b = \{(1.46H^{-0.8} + 1 - \rho_V/\rho_L)\exp(0.13E)[E - 0.2(1 - \rho_V/\rho_L)\exp(-0.35E)]^4\}^{-1}. \tag{20}$$

The heat fluxes computed on the basis of relations (18)-(20) are compared in Fig. 3 with experimental data obtained in the nonsteady cooling of preheated copper and steel pipes (inside diameter 4 to 20 mm, length up to 100 diameters) by a descending liquid-nitrogen flow. The scatter of the experimental points does not exceed ± 30%; i.e., it scarcely differs from the error of the approximation relations (18) and (19). This fact recommends the nomograph of Fig. 2 for the calculation of heat transfer associated with film boiling in pipes in the following ranges of regime parameters: $\mathrm{Re}_L = 8\cdot10^4$ to $1.5\cdot10^6$; $z = 7$ to 100; $\mathrm{Pr}_L = 1.9$ to 3.3; $P/P_{cr} = 0.06$ to 0.63; $k_L = 0.02$ to 0.95; $k_V = 0.9$; $\theta = c_V(T_W - T_S)/r \leq 9$.

NOTATION

A, constant in Eq. (1); B, constant in Eq. (2); $b = 2\delta/D$, dimensionless thickness of vapor film; c, specific heat at P=const; D, diameter of heat-transfer surface; E, dimensionless parameter in Eq. (8); $f(z/D) = (1 + 4\,\mathrm{Nu}_{L0}/\mathrm{Re}_{L0}\mathrm{Pr}_L\cdot z/D)^{-1}$; $f_1(z/D) = 1 + 1.22\exp(-0.038\,z/D)$; H, dimensionless parameter in Eq. (7); m, power exponent in Eq. (1); n, power exponent in Eq. (2); Nu, Nusselt number; $\mathrm{Nu}_L = q_L(D-2\delta)\lambda_L(T_S - T_L)$; $\mathrm{Nu}_V = q_W 2\delta/\lambda_V(T_W T_S)$; Pr, Prandtl number; q, specific heat flux; r, specific heat of vaporization; Re, Reynolds number; $\mathrm{Re}_L = u_L(D-2\delta)/\nu_L$; $\mathrm{Re}_V = u_V\cdot 2\delta/\nu_V$; T, temperature; U, velocity; z, axial coordinate; δ, thickness of vapor film; λ, thermal conductivity; ν, kinematic viscosity; ρ, density; σ, surface tension. Indices: L, liquid; V, vapor; s, saturation; w, wall; 0, entry.

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INFLUENCE OF HEAT FLUX AND PRESSURE ON HEAT TRANSFER ASSOCIATED WITH FULLY DEVELOPED NUCLEATE POOL BOILING

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UDC 536.248.2:546.291

Heat transfer in the pool boiling of helium is investigated experimentally. The dependence of the heat-transfer coefficient on the heat flux and pressure is determined for the fully developed nucleate boiling regime.

The problems of heat transfer in liquid helium hold a special place in the thermal stability analysis of composite conductors used in the coils of superconducting magnets. Accordingly, we have conducted a systematic investigation of the inlfuence of various factors on the heat transfer and critical heat fluxes associated with helium pool boiling. The main factors investigated in our work include the heat-flux density, the prior "history" of the boiling process, the saturation pressure, the finish of the heating surface, the material of the heat-transfer surface, and the orientation of the surface in the field of gravity. The experiments were conducted with plane working test sections of various materials shaped into a disk with a diameter of 25 mm or a square plate with a cross section of 30 × 30 mm. The pressure was held constant in each experiment, while the heat input to the heating surface was gradually increased or decreased to produce the appropriate variation of the temperature differential. An analysis of the experimental data shows that the dependence of the temperature differential on the heat flux can be multi-valued in the nucleate boiling of helium, indicating the existence of different boiling regimes. For each surface it is possible to discern the most fully developed boiling regime, wherein the heat-transfer rate attains its maximum value for the given conditions. In that regime, the relationship between the heat flux q and the temperature differential ΔT is described by the power law

$$q = C\Delta T^n. \tag{1}$$

As in the case of other liquids, the proportionality factor C depends on the saturation pressure, and the value of the exponent n can be taken as roughly constant for different pressures. Also, C and n vary appreciably with the surface conditions.

In the present article we give the results of a study of fully developed nucleate boiling of helium in the pressure interval $(0.33 \text{ to } 2.13) \cdot 10^5$ N/m^2 on only two surfaces, one of copper and one of aluminum. The surface of the copper section (square plate) was polished to class 13 purity [according to All-Union State Standard (GOST) 2309-68], and the finish of the aluminum surface (disk) corresponded to class 10. For the copper heat-transfer surface, we obtained data both for horizontal (heating surface facing upward) and for vertical orientation, while for the aluminum surface we used only the horizontal orientation. In experiments on a rougher aluminum surface, the value of the exponent n in Eq. (1) is 1.5 for all investigated pressures, which is in good agreement with the data of other authors for copper [1, 2] and platinum [3] heating surfaces. The dependence of the heat flux on the temperature differential on the polished copper surface in our experiments is described by the power law (1) with exponent n = 3.33. The relationship between the heat-transfer coefficient $\alpha = q/\Delta T$ and the heat flux in this case is consistent with the well-known dependence for ordinary liquids: $\alpha \sim q^{0.7}$.

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 33, No. 5, pp. 828-831, November, 1977. Original article submitted January 19, 1977.